

Interpreters

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Finite Maps

- empty map, with \emptyset as its domain
- $m(k)$ mapping of key k in map m
- $m[k \mapsto v]$ extension of map m to also map key k to value v

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$$\frac{}{m[k \mapsto v](k) = v} \quad \frac{k_1 \neq k_2}{m[k_1 \mapsto v](k_2) = m(k_2)}$$

Interpretation

Constants	n	\in	\mathbb{N}
Variables	x	\in	Strings
Expressions	e	$::=$	$n \mid x \mid e + e \mid e \times e$

$$\llbracket n \rrbracket v = n$$

$$\llbracket x \rrbracket v = v(x)$$

$$\llbracket e_1 + e_2 \rrbracket v = \llbracket e_1 \rrbracket v + \llbracket e_2 \rrbracket v$$

$$\llbracket e_1 \times e_2 \rrbracket v = \llbracket e_1 \rrbracket v \times \llbracket e_2 \rrbracket v$$

Substitution

$$[e/x]n = n$$

$$[e/x]x = e$$

$$[e/x]y = y, \text{ when } y \neq x$$

$$[e/x](e_1 + e_2) = [e/x]e_1 + [e/x]e_2$$

$$[e/x](e_1 \times e_2) = [e/x]e_1 \times [e/x]e_2$$

THEOREM 4.1. *For all e, e', x , and v , $\llbracket [e'/x]e \rrbracket v = \llbracket e \rrbracket (v[x \mapsto \llbracket e' \rrbracket v])$.*

A Stack Machine

Instructions $i ::= \text{PushConst}(n) \mid \text{PushVar}(x) \mid \text{Add} \mid \text{Multiply}$
Programs $\bar{i} ::= \cdot \mid i; \bar{i}$

$$\begin{aligned} \llbracket \text{PushConst}(n) \rrbracket(v, s) &= n \triangleright s \\ \llbracket \text{PushVar}(x) \rrbracket(v, s) &= v(x) \triangleright s \\ \llbracket \text{Add} \rrbracket(v, n_2 \triangleright n_1 \triangleright s) &= (n_1 + n_2) \triangleright s \\ \llbracket \text{Multiply} \rrbracket(v, n_2 \triangleright n_1 \triangleright s) &= (n_1 \times n_2) \triangleright s \end{aligned}$$

A Stack Machine

$$[n] = \text{PushConst}(n)$$

$$[x] = \text{PushVar}(x)$$

$$[e_1 + e_2] = [e_1] \bowtie [e_2] \bowtie \text{Add}$$

$$[e_1 \times e_2] = [e_1] \bowtie [e_2] \bowtie \text{Multiply}$$

THEOREM 4.2. $\llbracket [e] \rrbracket (v, \cdot) = \llbracket e \rrbracket v$.

Imperative Language

Constants	n	\in	\mathbb{N}
Variables	x	\in	Strings
Expressions	e	$::=$	$n \mid x \mid e + e \mid e \times e$
Command	c	$::=$	skip $\mid x \leftarrow e \mid c; c \mid$ repeat e do c done

$$\begin{aligned}f^0 &= \text{id} \\ f^{n+1} &= f^n \circ f\end{aligned}$$

$$\begin{aligned}[[\text{skip}]]v &= v \\ [[x \leftarrow e]]v &= v[x \mapsto [[e]]v] \\ [[c_1; c_2]]v &= [[c_2]]([[c_1]]v) \\ [[\text{repeat } e \text{ do } c \text{ done}]]v &= [[c]]^{[[e]]v}(v)\end{aligned}$$

Loop Unrolling

$${}^0c = \text{skip}$$

$${}^{n+1}c = c; {}^nc$$

$$|\text{skip}| = \text{skip}$$

$$|x \leftarrow e| = x \leftarrow e$$

$$|c_1; c_2| = |c_1|; |c_2|$$

$$|\text{repeat } n \text{ do } c \text{ done}| = {}^n|c|$$

$$|\text{repeat } e \text{ do } c \text{ done}| = \text{repeat } e \text{ do } |c| \text{ done}$$

THEOREM 4.4. $\llbracket |c| \rrbracket v = \llbracket c \rrbracket v.$